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MMS Project 13

Assessment and criticality of damage and defects in material systems

Task 5 Report

REVIEW AND IDENTIFICATION OF METHODS FOR ANALYSING THE EFFECTS OF DEFECTS AND DAMAGE

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This report forms deliverable M6/D1 of Task 5 of the MMS Project 13 ‘***Assessment & criticality of defects & damage in materials systems***’.

Contents

1 Introduction

A major output of the MMS13 project is to be a procedural guide for defect assessment in composite materials. This will be based to a large extent on analysis aimed at determining the acceptability of different defect types. The utility of the procedural guide therefore depends heavily on the analysis methods selected for inclusion. Where possible existing accepted and validated methods will be used and the focus of this report is on identifying methods that may be applicable.

This report considers published approaches for the analysis of a number of different defect and damage mechanisms. The objectives of this review are:

- to identify available approaches for the analysis of relevant defect and damage types;

- to assess the relevance of approaches in the context of fitness for service assessment;

- to identify what modifications may be necessary to facilitate practical implementation;

- to identify areas for which no suitable approaches are currently available.

The review will not attempt to consider all of the literature available in this area but will rather focus on that of direct relevance to the MMS13 project.

The approach to the review is to consider methods applicable to specific defect or damage types. In a detailed classification, over one hundred individual types can be defined. Many of these defect types, while perhaps different in detail, retain broadly similar morphology and effect on material behaviour. The majority of defect types can be adequately considered to fit into one, or sometimes more, of a smaller number of broad defect definitions, namely:

- Delamination
- Matrix micro-cracking
- Laminate through thickness cracking
- Material thinning
- Voids

The majority of in-service defects can be classified as either delamination or matrix micro-cracking hence these two types will be considered in detail here.

2 Delamination

2.1 Background

Delamination refers to situations in which failure (or inadequate adhesion) occurs on a plane between adjacent layers within a laminate. This type of failure is dominated by the properties of the matrix and since matrix strengths and toughness tend to be relatively low, laminated composites are prone to the development of delaminations. In many types of composite structure (e.g. aircraft, marine) delaminations are the most common form of defect/damage.

Delaminations can have a wide variety of causes – these are summarised briefly below.

Impact damage often gives rise to local delamination. Even though the impact may be transverse to a laminate, damage will often propagate significantly along the planes between adjacent layers since these planes do not benefit from the reinforcing effect of fibres. Hence sites of impact often form zones of interlaminar splitting. Within these delaminated zones there may also be other damage types such as matrix micro-cracking and fibre breakage. Growth of the defect (this often being a concern from a fitness for service perspective) will in many cases occur however by extension of the delamination into the surrounding material that may not have been directly affected by the initial impact event.

Cracking (matrix-microcracking) in the through thickness direction within individual layers of a laminate can give rise to delaminations [1]. Such cracking tends to occur perpendicular to the fibre direction within individual layers and extends through the entire thickness of the affected layer. Once a crack has developed, some opening of the crack is possible near the middle region of the affected layer but the crack tips are restrained by their respective adjacent layers (which may have considerably stiffness). In response a multi-axial stress state develops at the crack tips, often with strong through thickness tensile components. These tensile stresses may be sufficient to initiate interlaminar failure from the crack tips [2, 3]. Hence as the microcracking density is increased so the delaminated area increases. As with impact induced delamination, this type of damage will typically involve more than one mechanism but the delamination may be that which ultimately determined serviceability.

Through thickness failures arising from overload or fatigue can also develop into delaminations. Laminated composites tend to be weak in the through thickness direction and consequently are sensitive to loads giving rise to stresses in the through thickness direction. Consideration of this aspect of the nature of laminated composites is essential in arriving at successful design solutions but situations in which through thickness failures occur must still be considered. The stresses of concern here are direct through thickness tension and through thickness shear. Delaminations occurring under the action of these stresses are often not associated with other damage types developing simultaneously. It is worth noting that the stresses giving rise to delamination may be small in magnitude (especially by comparison to typical laminate in-plane stress allowables) and often exist as secondary effects [4].

The direct stresses referred to above arise from a combination of the structural geometry and the applied loading [4-6]. Through thickness stresses may also be developed at the edges of laminates, even when no externally applied through thickness tension or shear loads exist [7, 8]. In laminates containing adjacent layers with different elastic properties compatibility requirements at free edges mean that through thickness stresses exist even under the action of purely in-plane loads. The magnitude of these stresses depend strongly on the materials and laminate construction. The resulting stresses will usually be smaller than the applied in-plane stresses with which they are associated but since they act in a direction that is usually relatively weak, static or fatigue failures, giving rise to delaminations can arise.

Another source of delaminations is inadequate bonding between layers. This is usually associated with manufacturing deficiencies (e.g. prepreg backing not removed or improper wet out locally) but may only become a concern after growth accumulates over a long period in service.

2.2 Growth and failure of delaminations

The above discussions refer to the initiation of delaminations. From a fitness for service point of view, growth is an equally, and in many respects a more, important consideration. Growth of delaminations is governed by the conditions at the delamination front. The growth mechanisms are very complex when analysed in detail, with one of the main difficulties being that the region in which the mechanism is active is so small as to be very much on the scale at which material heterogeneity must be considered [9].

Growth of delaminations should, in the context of fitness for service assessment, be considered from two points of view, as discussed below.

Firstly, it is important to establish if growth can occur, i.e. the concept of fatigue threshold conditions applies here. The importance of a threshold is obvious – if the delamination operates below the threshold condition then growth is not expected and, unless there is some other immediate impact on fitness for service, continued operation is readily justifiable.

Secondly, if it is established that growth can occur, i.e. operation beyond the threshold condition, then justification for continued service relies on

(i) the ability to define the critical condition, i.e. the characteristics of the delamination at such time that it renders the structure not fit for service

and

(ii) the ability to predict or monitor ongoing growth so that there is a sufficient degree of confidence that intervention can be specified before the critical condition is attained.

Since neither of the above are trivial considerations, the effort required in justifying fitness for service will often be significant when the threshold conditions for growth are exceeded.

The critical condition referred to in (i) above is often taken as defined by unstable fracture at the delamination front. It is worth nothing that this is often not a critical condition in the sense that would normally apply to fracture in steel structures. Unstable delamination growth does not always lead to catastrophic failure in a composite structure and propagation to a point at which stability is re-attained is sometimes possible. Nevertheless, the unstable fracture condition remains practically very important and has been the subject of a great deal of research.

Delamination is generally regarded as being governed primarily by fracture mechanics rather than strength of materials considerations [9]. Fracture mechanics considers that failure is governed by an energy criterion relating changes, for an incremental extension of crack face area, in the work of the applied loads and internal strain energy to a material toughness parameter. The criterion can be expressed as

$$\frac{d}{dA}\left(W-U\right)=G_c \tag{1}$$

where A represents crack face area, W is the work of the applied loads, U is the strain energy and G_c is the material fracture toughness (this being the energy required for a unit increase in crack surface area). The parameter $d/dA\left(W-U\right)$ is referred to as the applied strain energy release rate and is usually assigned the symbol G.

There are some important restrictions to the conventional fracture mechanics approach that should be considered in its application to delaminations. These are discussed below.

(i) The toughness parameter, G_c, is a property of the material that is only uniquely defined when stress-strain behaviour remains everywhere linearly elastic. Hence approaches in which this is assumed to be the case are referred to as incorporating linear elastic fracture mechanics (LEFM). The assumption of linear elasticity applying everywhere is one that strictly has limited applicability. In the case of metals, yielding tends to occur at higher stress levels and the crack tip (i.e. the region of primary interest) is in a zone where stress-strain behaviour is distinctly plastic. Nevertheless, LEFM has been shown to be a practically useful predictive tool in such circumstances, albeit with minor modifications to account for plasticity being incorporated in certain cases [11].

The situation in composite materials is complicated by the presence of different constituent materials with distinctly different elastic properties and strengths. Nevertheless, it has been shown that in composites a damage zone forms in the region of the crack tip and this, broadly, has effects similar to the plastic zone at the crack tip in a metal. Material behaviour in the damage zone may not be linearly elastic but, as with metals, the net effect is often not such as to limit the practical utility of the LEFM approach [12].

(ii) The uniqueness of the toughness parameter also depends on self-similarity in crack geometry and failure processes, i.e. the conditions on both sides of the crack must be the same and the failure process should not change with small increases in crack depth. These conditions will not normally restrict the applicability of fracture mechanics to homogeneous and isotropic materials such as metals. They are, however, potentially very important when applying fracture mechanics to composites since it is evident they will not widely apply. For example, in laminates undergoing delamination the material on either side of the delamination can be very different, e.g. a unidirectional layer adjacent to a woven layer. Furthermore, delamination usually involves a number of distinct failure processes in the zone of the delamination front, e.g. matrix microcracking, fibre debonding and fibre bridging. In addition, on the scale of the damage zone ahead of the crack front, the dominant damage mechanism may continuously change (in response to localised changes in composite morphology) as the crack advances. In these circumstances a unique toughness parameter can not strictly apply.

The above consideration suggests that the conventional fracture mechanics approach is not always theoretically consistent in application to the analysis of delaminations. This may indeed be true, however, the fracture mechanics concept of crack growth being governed by stability considerations as determined by changes in energy remains applicable. Hence, the conventional fracture mechanics approach as applied to delaminations is by nature a tool for engineering approximation rather than a basis for accurately describing and predicting the failure processes in detail. It is important that the fracture mechanics approach be recognised as such, even when applied to fitness for service assessment where its limitations may not always give rise to practical difficulty.

Equation 1 indicates that implementation of the fracture mechanics approach relies on estimation (typically by analytical or numerical techniques) of the applied strain energy release rate

$$G = \frac{d}{dA}\left(W - U\right) \tag{2}$$

and comparison to the toughness parameter G_c (which is usually established by material tests). It is convenient, in simplifying both analysis and test requirements, to decompose the applied strain energy release rate and the toughness into parameters corresponding to three possible modes of cracking. Three strain energy release rates (G_I, G_{II} and G_{III}) and three toughness parameters (G_{Ic}, G_{IIc} and G_{IIIc}) are defined as corresponding to cracking modes I, II and III respectively. Mode I considers crack opening, Mode II considers shearing and Mode III considers tearing (see Figure 1).

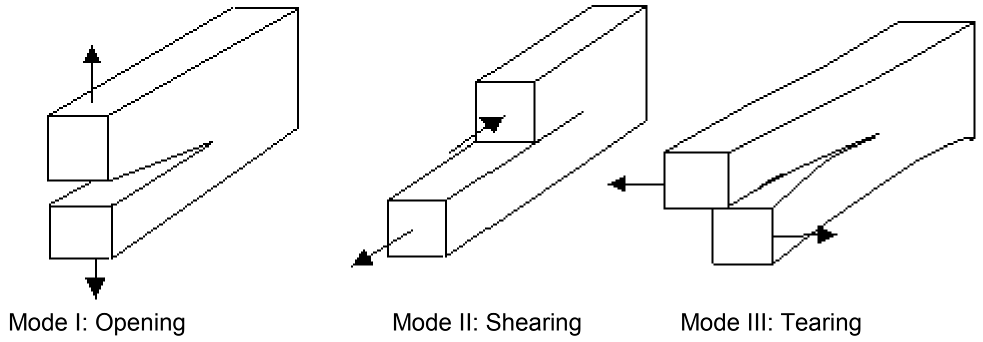

Figure 1: Possible modes of delamination growth

In most practical cases of delamination it has been found that only modes I and II are active and mode III applied strain energy release rates are negligible [9].

In some specific cases delamination may be strongly dominated by one mode with the other playing a negligible role. In such cases direct comparison can be made between the strain energy release rate and toughness parameter for the dominant mode. However, in general, consideration must be given to mixed mode loading and growth. In mixed mode cases, it has been shown that the failure condition for each mode cannot be regarded as independent of the energy release rate of the other [12]. Hence equation 1 must be expressed as an interactive criterion. Numerous criteria, many of which are quite different in approach, have been proposed. Reeder [13] performed a review of available criteria in 1992 and concluded that much remains to be understood about the phenomenon of mixed mode fracture. Nevertheless, while it is apparent that no rigorous theory explaining the intrinsic nature of interaction exists, several empirical approaches that offer satisfactory estimation of measured behaviour are available. The power law approach, see for example Russel and Street [14], has been shown to reasonably accurately predict behaviour across a range of cases and is now widely applied. This power law criterion can be expressed as

$$\left(\frac{G_I}{G_{Ic}}\right)^{\alpha} + \left(\frac{G_{II}}{G_{IIc}}\right)^{\beta} = 1 \tag{3}$$

Note that in order to use the criterion the empirical constants α and β must first be determined. These should strictly be established by performing mixed mode testing on the material under consideration. It is apparent however that in many cases of delamination in laminates consisting of unidirectional layers or where laminates are bonded together using an adhesive interface, both α and β are approximately one [9, 15]. There is insufficient evidence to suggest that these values can be assumed for α and β in other types of material and there are reported results showing that $\alpha = \beta = 1$ is not an accurate representation in some cases [13]. Nevertheless, analysis of a range of test data does suggest that taking $\alpha = \beta = 1$ may represent a conservative, or only marginally un-conservative, approach in many cases [12, 13, 15, 15], i.e. it appears unusual for α or β to be significantly less than one.

It is worth pointing out here that the power law criterion, like all others proposed, provides in essence a method for extrapolation from measured data by means of a curve fit. The nature of the fitted curves appears to be determined more by what is mathematically convenient (e.g. obtaining a reasonable correlation with a minimum number of experimentally derived constants) than by a sound physical basis for a particular nature of interaction. For this reason caution should be exercised in applying these criteria in situations for which test data is not available. This, and the limited data available supporting the view that $\alpha = \beta$ will usually be conservative in the power law criterion, suggests that additional conservatism may be required in cases where validation by mixed mode testing is not possible. Such conservatism might be introduced in a variety of ways, including the following.

- Use of $\alpha < 1$ and/or $\beta < 1$
- Taking the right hand side of equation 3 as less than one
- Application of margins to the G_{Ic} and G_{IIc} values in equation 3.

Further investigation will be required to establish the best approach, however, the use of margins on material toughness offers some benefits and is consistent with conventional fitness for service approaches.

Clearly, whatever mixed-mode criterion is selected, its implementation will rely on estimation of the strain energy release rates for the applied loading and knowledge of the toughness parameters. The toughness parameters are material properties that will usually be established by mode specific tests. Testing is covered elsewhere in the MMS 13 project but it is sufficient here to say that standard test methods for Mode I and Mode II have been defined that standard test methods for Mode I and Mode II have been defined and agreed following several round-robin exercises. The relevant test methods are relatively straightforward, using a double cantilever beam (DCB) specimen for Mode I and an end notch flexure (ENF) specimen for Mode II. The costs associated with such tests are relatively small and data can be rapidly generated.

The above should not however be taken to suggest that a defect assessment approach must always rely on tests conducted on the material in which the defects exist. There will, for example, be many situations in which it not possible to obtain suitable representative material or manufacture similar materials for testing. Furthermore there may be cases where the defects are not so severe as to justify an assessment requiring testing. In such cases what is needed is that the properties used be such as to ensure conservatism. Hence a defect assessment procedure should make allowance for the use of representative generic properties. An important requirement of the assessment procedure in these circumstances is to provide guidance to ensure that the properties selected and are always estimated in a manner giving rise to a conservative assessment. At the same time the degree of conservatism must not be so excessive as to limit the utility of the procedure.

The use of generic properties is a feature of defect assessment procedures for metallic structures but their definition can be expected to be more challenging for composite materials as a result of the large number of different materials available. An attempt at defining ‘lower bound’ toughness values for different material classes may be worthwhile however. Successful implementation of such an approach will rely heavily on the way in which material classes are defined. An important requirement here is to define classes so as to capture fundamental differences in properties but at the same time restrict the number of classes.

Perhaps the most challenging aspect of delamination analysis lies in the estimation of the applied strain energy release rates, namely G_I and G_{II}. These depend primarily on the nature of the applied stress state and the crack geometry. Analytical solutions are available for some simple stress states and geometries. These typically consider one dimensional geometries and loading, e.g. a beam of constant thickness and width with a full width delamination with no change in loading across the crack front. There are some limited practical cases where such analytical solutions do apply and for these cases definition of the assessment route will be fairly straightforward.

For most general geometries and loading likely to be encountered in practice it is apparent that no widely applicable analytical solutions exist, even where these might be approximate. A variety of numerical techniques have been applied to estimating strain energy release rates but it is evident that the finite element method is now the most widely used [9]. The advantage of the finite element method is that it imposes no inherent restriction on geometry, applied stress state and material behaviour. The approach to estimating strain energy release rates using finite element methods was initially to evaluate the W-U term in equation 2 for two models, the second considering an incremental growth in crack surface area. This approach proved useful in certain cases but difficult in general application, the main problem being that a G value for the structure as a whole is obtained with no information on its distribution along the crack front nor on the relative magnitude of the individual modes. Hence, except in some specific cases, it is not possible to apply the results in a failure criterion requiring direct comparison of strain energy release rate with material toughness. An alternative approach, still in reasonably common use, is to calculate G by integration of energy along a contour around the crack front, i.e. determination of the J-integral. This has the benefit of allowing estimation of a local strain energy release rate and evaluation for each mode. However, it requires the use of special collapsed elements at the crack front and this can add significantly to mesh and hence model complexity. A further problem is that it is not strictly applicable to cases where dissimilar materials on opposing sides of the crack tip give rise to an oscillating singularity.

In response the above difficulties, several alternative approaches have been developed in application of the finite element method. One of the most useful is the virtual crack closure technique [10]. This makes some assumptions regarding possible changes in the stress state for an incremental closure of the crack hence a single model can be used for estimating strain energy release rates. A major benefit of the method is that it allows accurate estimation of strain energy release rate while using a relatively simple mesh (although a fine mesh may still be needed at the crack tip). Furthermore, it allows separate calculation of G_I and G_{II} by isolating the relevant stresses for inclusion in the integrations needed to evaluate the parameters in equation 2. Further benefits include (i) the calculation of these values on an element by element basis and provision of results on a node by node basis and (ii) straightforward treatments of oscillatory singularities in stress.

One of the main problems remaining with application of the 3-D finite element method to delaminations is that a very fine mesh is normally required in the region of the crack front. This presents both a theoretical and practical difficulty. The theoretical concern commonly raised is that the element size in the region of the crack front often has to be selected so small as to be very much on the scale at which the material can not be treated as homogenous. This issue is, justifiably, the subject of debate in the research community however it is apparent that it does not, in itself, impose a significant restriction on the utility of the method as a tool for engineering approximation.

The practical difficulties associated with the need for a very fine mesh include the following.

(i) Models can be time consuming to generate.

(ii) Analysis tends to be highly computationally intensive.

(iii) Convergence studies are usually needed to verify that the mesh is adequate and these are in themselves time consuming.

(iv) A high degree of skill and experience is needed on the part of the analyst to ensure a valid result.

The above are all of concern from a fitness for service assessment perspective. Most practical defect assessment procedures allow a phased approach to assessment and this type of approach will be appropriate for the guidance to be provided by MMS 13. Assessment begins with relatively simple, rapidly implemented methods (that will usually necessarily incorporate greater conservatism) and moves through to more sophisticated and complex methods (where conservatisms will typically be lowered). Given the requirements for implementation of the finite element method to delaminations, it is clear that it is suitable only for higher level assessments and alternative approaches will have to be found for the lower levels. Definition of these alternatives represents a significant challenge when analytical solutions are not available for most geometries of interest.

One alternative approach that is considered to have potential is the use of solutions for simple geometries to provide conservative estimates of strain energy release rates for more complex geometries that have at least some similarity. Solutions for delaminations in beams are available for a wide variety of cases, see for examples [17-20].

There are situations in which use of the beam solution for a plate delamination is clearly conservative. Consider for example a finite length delamination centrally located in a composite beam subject to a bending moment as shown in Figure 2. The bending moment introduces a constant applied strain energy release rate (Mode II only) across the delamination fronts.

The beam solution contains two main conservatisms compared to the situation for a plate with a rectangular delamination.

Firstly, the full moment must be transferred across the delaminated region in the beam whereas in the plate the moment will tend to be preferentially carried by the unaffected material at the plate edges since this retains its full stiffness. In simple terms this amounts to applying a margin or partial safety factor to the applied loading.

Secondly, restraint on rotation and sliding at the crack front in the beam is depends on the local bending and shear stiffness of the material. In the plate case further restraint is imposed by the unaffected material the edges of the delamination front hence for a given far field loading the deformations will be smaller in the plate case and the strain energies correspondingly also smaller. Hence the applied strain energy release rate will tend to be less – again this has the similar effect to applying a margin or partial safety to the applied load.

Both of the above considerations also apply to a plate with a curved, e.g. circular or elliptical delamination and the restraint imposed on deformations will tend to be greater in these cases.

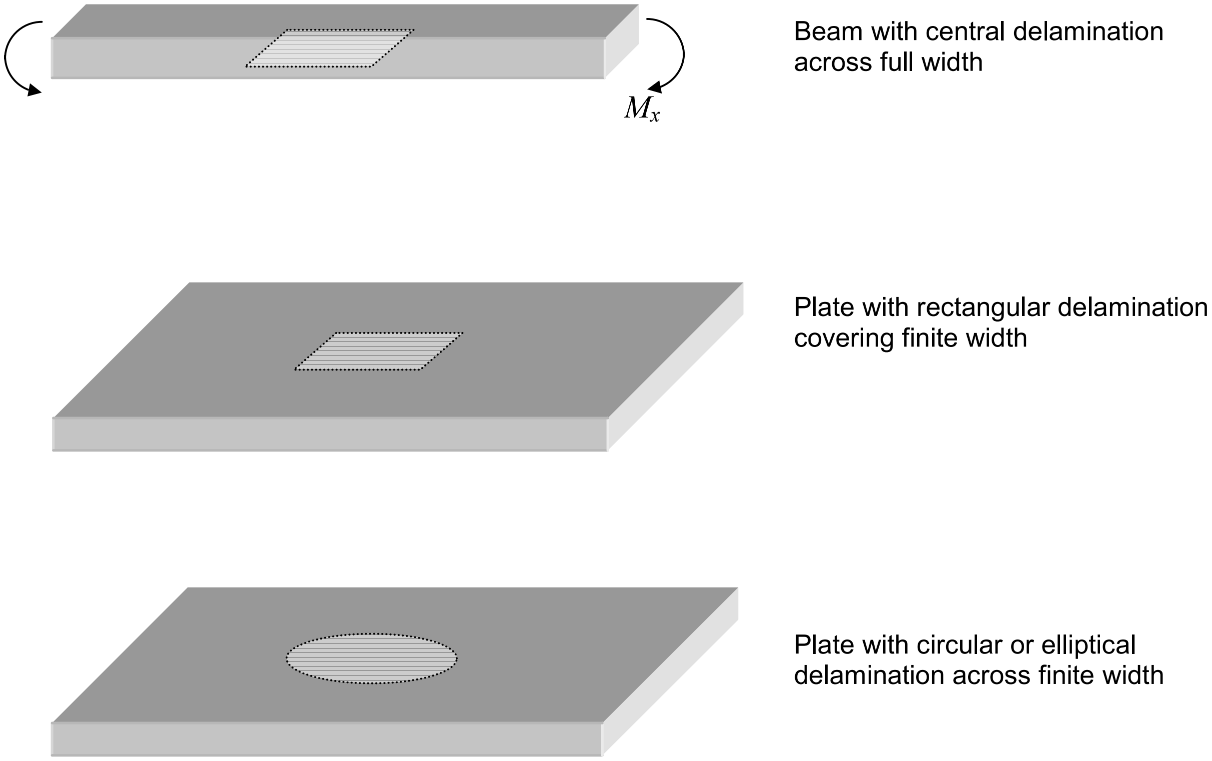

Figure 2: Beam and plate delaminations

There may be some complicating factors associated with the edges of a rectangular delamination and the effects of multi-axial loadings but the above simple example illustrates that it is possible to demonstrate in particular cases that the beam solutions are indeed conservative. Clearly however the degree of conservatism is important as this directly affects the utility of the approximation. The solutions will be of no value if they overestimate applied strain energy release rate to such an extent that justification of fitness for service is rendered impossible for defects of any measurable dimension. At this stage it is not possible to comment on the extent of conservatism that is likely to arise for different situations. However, it is recommended that comparisons with the chosen beam solutions for a range of cases be made with the results of detailed finite element analyses on the actual geometry. This will show the extent of conservatism and indicate the situations for which steps need to be taken to address it. It is possible, for example, that reasonable correction factors could be determined by simple empirical functions of some parameter relating to the differences between the beam and plate geometries, e.g. ratio of delamination with to plate width, curvature of the delamination front relative to the plate thickness.

The preceding discussions have considered the critical condition for delamination growth. Growth of delaminations can also occur at sub-critical conditions by fatigue type processes, with a small amount of growth occurring under each fatigue cycle. There is however a threshold loading range below which growth does not occur. For a single mode loading condition this is defined by a threshold strain energy release rate [21,22] that can be considered as an inherent material property, albeit one that depends on where in the laminate the delamination occurs. Growth occurs when the range of the applied strain energy release rate, i.e. maximum – minimum, for a stress cycle exceeds the threshold. Tests can be done to determine the threshold values but these are necessarily more time consuming and expensive than those to determine critical strain energy release rates.

Published research into defining threshold conditions under mixed mode loading appears limited. Experimental results [25] suggest that there is a strong interaction between the threshold conditions for modes I and II. Further experimental data, to be generated as part of the MMS 13 programme, will have to be reviewed before arriving at a conclusion on this subject but it appears that a similar interaction criterion to that used for the critical conditions (see equation 3) may be appropriate. Again the constants in the equation would have to be experimentally determined but it may be possible to define generic lower-bound values in making for practical implementation.

Once beyond the threshold conditions, sub-critical growth rates have been found to vary exponentially with the range of applied strain energy release rate [21-27]. For single mode loading an empirical Paris Law type approach has been found to be effective in relating crack growth to strain energy release rate, i.e.

$$\frac{da}{dN} = A\left(\Delta G\right)^{m} \tag{4}$$

where *da/dN* is the amount of crack growth for a fatigue cycle which has a range in strain energy release rate of ΔG. The constant A and exponent m are determined experimentally for the material under consideration. There is evidence that the ratio of maximum to minimum G value during the cycle does have an effect on growth rate, see as examples [21], [28-29]. It is also apparent that there can be a frequency effect [30], i.e. the growth per cycle can be influenced by the frequency of the loading. Hence, care should be taken when defining A and m for use in equation 4 to ensure that the values used will be conservative.

It is worth noting that the difference between the critical strain energy release rate and threshold strain energy release rate tends to be relatively small for delamination and debonding in composites [21, 23, 31]. Published data suggests that the ratio is unlikely to be more than ten. This ratio is important in that it determines to a large extent the life of delaminated structures under fatigue loading. A large ratio will generally mean that reasonably long lives can be attained (making for potentially straightforward safe management of the structure by implementation of an inspection and monitoring plan). A small ratio, on the other hand, can result in very short life after the initiation of delamination growth. This is made worse by the often inherently poor resistance to delamination fatigue in composites, i.e. A and/or m in equation 4 tend to be relatively high. The resulting life can be so short as to render management by inspection and monitoring impossible, i.e. a delaminations greater than the threshold size for growth tend to grow so rapidly that failure occurs so quickly that it could not have been detected within any practical inspection interval.

This situation is particularly relevant to bonded joints and has been considered more in the literature in that context than in respect of delaminations in composites. Nonetheless the two are comparable and the debate in the literature is highly relevant to delaminations. It has, for example, been strongly suggested [31] that the use of fracture mechanics as part of a design and operational strategy, while very successful for mechanically fastened (typically metallic) structures, has little chance of success when applied to bonded joints. The foundations for this argument are the low ratio of critical to threshold toughness and inherently large growth per load cycle. Others have argued that fracture mechanics does have a role to play in the design of bonded joints, see for example [15], provided the material behaviour is well understood. The former approach places more emphasis on sound up-front design and manufacturing practice with a focus on designing to ensure substantial defect tolerance, i.e. such that all conceivable defects are smaller than the threshold size. It would suggest that there is little utility in a fitness for service type approach to delaminations under fatigue loading, except to ensure that the acceptable size limit is set to below the threshold size.

Regardless of the merits of the various arguments, the use of the threshold as a limit is likely to offer some benefits for practical fitness for service assessment. The threshold size for growth is a sensible limiting size for a preliminary (low-level) assessment. It may be possible to define a successful management strategy for defects larger than threshold size but this will always require significant additional engineering assessment (which may include detailed modelling and material characterisation). Such additional effort will only be justifiable in specific cases and hence should only be provided for in the highest level analysis of the fitness for service procedure.

In practice, delamination fronts are often subject to mixed mode loading and equation 4 does not apply directly. There is at present, based on literature reviewed, no generally accepted approach to estimating fatigue growth under mixed mode conditions. It has been shown that an adaptation of the Paris Law, namely

$$\frac{da}{dN} = A_I \left(\Delta G_I\right)^{m_I} + A_{II} \left(\Delta G_{II}\right)^{m_{II}} \tag{5}$$

results in a good fit to experimental data for specific cases [22]. Note however that A_I, A_{II}, m_I and m_{II} are dependent on the mode ratio, i.e. $\Delta G_I / \Delta G_{II}$, hence equation 5 relies on generation of loading mix specific parameters rather than on the parameters determined from a single mode test. The nature of mixed mode growth has been found to be very complex, with a number of regimes existing over which different behaviour occurs, see for example [32]. Published literature provides little at present that can assist in a fitness for service approach in this regard except to suggest that the relevant behaviour would have to be established by test. This situation might be regarded as unsatisfactory but it should be recognised that growth beyond threshold is only likely to be a concern for situations where extensive engineering and test work would be justified in any event.

3 Matrix Microcracking

3.1 Background

Matrix microcracking refers to intralaminar or ply cracks that traverse the thickness of the ply and run parallel to the fibres in that ply. Their existence does not necessarily mean catastrophic failure of the composite as they can be present only in certain plies (usually those transverse to the main loading direction) and while the fibres (which carry most of the load) remain intact. Matrix microcracks can develop under tensile loading, fatigue loading, thermal loading and impact conditions. They sometimes arise in composites during manufacture but are more commonly associated with in-service effects.

Matrix microcracking is one of the most common forms of damage encountered in composite materials and is often a precursor to overall failure. Hence it is important that a defect assessment procedure has the ability to estimate the effects of microcracking on composite properties and also define tolerable limits for cracking.

3.2 Initiation of microcracking

Matrix microcracks can initiate under a number of conditions. While initiation is not of direct relevance to defect assessment (which considers defects already present) it is useful to briefly consider initiation mechanisms and associated theoretical models. A detailed treatment of matrix microcrack initiation is presented in [33]. An important observation in experiments concerned with microcrack initiation is that it is not directly related to the transverse tensile strength of the affected plies. Results for a variety of materials show that the thickness of the plies subject to transverse tension is an important parameter (see as examples [34] and [35]). This suggests that the initiation of matrix cracking is governed by fracture mechanics considerations. In most cases of microcracking, growth is, at least from a practical point of view, instantaneous across the thickness of the affected ply (or plies). The adjacent layers, which will typically have fibres at a different orientation to that in the affected layer, usually act as crack stoppers. Hence these layers often restrain any further through thickness growth.

This type of crack growth, being unstable at effectively zero dimension but growing instantaneously to a finite dimension (at which stability is retained), is unusual from a traditional fracture mechanics perspective. The stability considerations in a conventional fracture mechanics approach are based on an infinitesimal growth in crack surface area (as indicated by the differentiation with respect to area in equation 1), but the crack necessarily begins at finite dimension. The requirements of properly dealing with this difference have given rise to a branch of fracture mechanics now referred to as finite fracture mechanics [36-37]. The principles are essentially the same but the energy criterion relates the total amount of energy released in the formation of a completely through ply microcrack to a critical strain energy release rate for the later containing the crack.

Application of finite fracture mechanics relies on (i) estimation of the energy released in the formation of cracks and (ii) knowledge of the critical strain energy release rate for microcracking. The energy released in the formation of cracks is determined by the change in the work of the applied loads and the internal strain energy. The former is usually a relatively straightforward consideration and is zero in the case of displacement controlled loading. Estimating the change in strain energy is more complex and relies on knowledge of the internal stress and strain state before and after the formation of the cracks. When considering initiation, the pre-cracked stress and strain state can often be readily determined by application of a conventional laminate stress analysis approach and can usually be assumed independent of position within the layer. A potentially important complicating factor is that residual stresses should strictly be considered as these can have a significant effect in certain cases [33].

The stress and strain state after the formation of a crack (or cracks) is much more complicated with variation according distance away from the crack face and also through the thickness of the layer. In addition there is a change in the stress and strain states in all the remaining layers within the laminate.

A great deal of research effort has focussed on predicting the post-cracking stress and strain state to enable the calculation of strain energy. The approaches vary in complexity and accuracy but must attempt to describe, as a minimum, how the stress perpendicular to the direction of the cracking varies from zero at the crack face (an obvious boundary condition) to a finite value some distance away from the crack face. This depends on the nature of load transfer between the cracked ply and adjacent plies. The simplest models adopt a shear lag type approach in which the rate of increase in stress away from the crack face is related to the shear stress (or strain) on the interface between the cracked layer and adjacent plies. This allows a straightforward equilibrium equation to be set up describing the load transfer across the interface. Compatibility requirements are then used to define the relationship between interface shear strains and layer axial strains. When substituted into the equilibrium equation this results, in its simplest form, in an easily solved second order differential equation in a single variable, see as examples [38] and [39]. In this simplified one dimensional implementation, the shear lag approach has a number of shortcomings but it has been successfully extended to two dimensions by a number of researchers. Details on shear lag approaches are provided in a comprehensive review [40]. It is evident that in two dimensional form the shear lag approach is able to accurately predict stress and strain states after crack formation and is therefore a useful tool in the prediction of microcrack initiation by finite fracture mechanics.

A number of alternative approaches to the shear lag models have also been developed. These can be broadly categorised into those considering the equilibrium differential equations (and associated compatibility conditions) and those using energy methods.

A variety of approaches have been proposed to developing the equilibrium differential equations and significant contributions to this area have been made in work performed by McCartney at NPL, see for examples [41]-[44]. McCartney has developed load transfer models based on a comprehensive set of equilibrium differential equations and compatibility conditions that allow a close physical representation of the real mechanisms. The models rely on a minimum of assumptions regarding stress and displacement states and the solutions allow exact satisfaction of the equilibrium and interface conditions. Fracture mechanics concepts – necessary to estimate crack initiation and growth – have been incorporated and the models have been found to be good predictors of observed behaviour.

Energy methods, or variational principles, have also been successfully applied to estimating the stress and strain states in cracked laminates, see for a discussion [33] and [45]. The energy method approaches determine the equilibrium condition by minimisation of either total potential energy or total complementary energy. In the former approach a deformation state, defined by a finite number of unknown variables, is solved for by minimisation of the total potential energy with respect to each of the variables. The latter approach is similar except that a stress state is assumed and solution is by minimisation of the total complementary energy with respect to variables defining the stress state. It is worth noting that, in general, the energy approaches offer only approximate solutions to the deformation or stress state and the accuracy is related to how the relevant state is defined by the problem unknowns. In this respect the approach of McCartney, as mentioned above, may be more robust in implementation. Nevertheless energy approaches have formed the basis for analyses capable of accurately predicting experimentally observed initiation [33].

At present, models for accurately predicting microcrack initiation are restricted to consideration of a two dimensional stress state in the affected ply. Changes in both the direct stress in the direction of the fibres (i.e. parallel to the microcrack direction) and the in-plane shear stress are not included in the models. This, in effect, restricts application of the models to cases where the applied load is predominantly perpendicular to the fibre direction of the layer in which initiation is expected. Clearly this covers the important case of cross-ply (i.e. 0/90 type) laminates in which the primary loading is in the direction of the 0° plies. However, the analysis can not be readily extended to other loading types nor does it generally apply to angle ply laminates. There are two difficulties in extending the theory to these cases. Firstly, estimation of the stress and strain states in the presence of cracks is more complex. Secondly, the fracture mechanics criterion for the onset of cracking has to consider mixed mode loading on the crack front. This means that a suitable mixed mode fracture criterion is required, along with two critical strain energy release rate values. While it is likely that these issues will be addressed in future it can be expected that the complexity of the resulting models for crack initiation will be greatly increased.

3.3 Growth of microcracking

Growth of microcracking under static loading occurs by a similar fracture phenomenon to initiation. The only fundamental difference is that when considering growth the change in applied strain energy release rate is calculated from an already cracked state whereas initiation considers the changes from an uncracked state. Hence, while the starting stress state when considering growth may be more complex, the same basic analysis approach used for initiation can be applied.

The results of an initiation analysis based on fracture mechanics will give a stress at which cracks first appear and an average crack spacing at this stress. The crack spacing is an inherent result of the analysis in that the energies calculated are finite and associated with corresponding finite volume of material, the length of which defines the average crack spacing.

When a crack initiates, the stress at the crack face becomes zero but increases with increasing distance away from the face. As the applied load (or displacement) is increased, so the stress, at some distance from the crack face may become such as to initiate a further crack (or cracks). Again the stress at the new crack faces is reduced to zero and a further increase in overall load (or displacement) is needed before further cracking initiates at some distance from the new crack faces. In the analysis, each of these events can be estimated using the same basic approach and models can accurately predict microcrack density as a function of stress, see for examples [33] and [45]. Once beyond the load for initiation, the crack density tends to increase quite rapidly at first with applied load but as more cracks form so the rate of increase in crack density with applied load tends to decrease. This is a result of the applied strain energy release rate (for a given applied load) decreasing with increasing crack density. Such behaviour is to be expected since each new crack means there is a smaller volume in which energy released on formation of the next crack is stored. Hence a higher energy input (in terms of the applied load) is needed to form the crack.

Matrix microcrack growth can also occur under cyclic fatigue type loading. Under fatigue conditions, crack accumulation is influenced by the applied loading and the number of cycles. It has been shown that cracking behaviour under fatigue loading is governed to a large extent by the range of applied strain energy release rate for each fatigue stress cycle. An adaptation of the conventional Paris Law type approach has been found to describe crack accumulation across a range of conditions [33]. In this adaptation (which is essentially an application to finite fracture mechanics), the rate of increase in crack density is related to the range of applied strain energy release rate, i.e.

$$\frac{dD}{dN} = A\left(\Delta G\right)^m \tag{6}$$

where D is crack density, N is the number of cycles, A and m are material specific parameters (usually established by testing) and ΔG is the range of applied strain energy intensity. The key difference from the conventional Paris Law is that a crack density is used rather than a crack dimension. It is also worth pointing out that ΔG is a function of the applied loads and crack density whereas in the conventional Paris Law ΔG is a function of applied loads and crack dimension. The dependence on crack density clearly means that implementation of equation 6 relies on application of the finite fracture mechanics methods mentioned in the section above on initiation. Hence analytical estimation of crack density accumulation under fatigue conditions is relatively complex.

Crack density can also increase under long-term static loading. It has been shown that the evolution of crack density under conditions of long term static loading can be described approximately by

$$\frac{dD}{dt} = A\left(\Delta G\right)^{m} \tag{7}$$

The parameters for equation 7 have to be determined experimentally.

3.4 Practical application of microcracking analysis to integrity assessment

Available methods for microcrack analysis offer a useful theoretical description of the underlying processes. However, the theory is complex and implementation relies on knowledge of a number of material parameters that are not readily established. Hence, practical application is not straightforward and some important assumptions or simplifications may be needed. Nonetheless, approaches based on crack density (or spacing) have been proven to work in practice and used as a foundation for long-term design methods [46].

An important concern is that estimation of material properties by microcracking analysis relies on knowledge of crack density. Hence, its application to integrity assessment of a real structure would depend on knowledge of the crack density. This is not readily measured in most cases hence estimates, probably based on some sort of indirect measurement, would be required. Given this concern it appears at present that the theory will not be directly applicable to practical defect assessment. However, the underlying principles and descriptions of the physical processes can form the basis for simplified models and definition of test approaches that can be used in defect assessment. Implementation in a practical defect assessment procedure will therefore require further work on synthesis of available theories.

Given the difficulty in non-destructively measuring crack density a conservative starting point is to assume that that microcracking is extensive over the affected region. This means that the elastic properties of individual layers should be adjusted assuming that the matrix is fully cracked, i.e. in the worst case both the transverse Young's modulus (E_2) and the in-plane shear modulus (G_{12}) should be reduced to zero for the affected layers. This will allow the elastic properties for the damaged laminate as a whole to be estimated along with updated ultimate strengths.

Depending on the types of materials and laminate constructions being considered the above approach may be overly conservative where the region of damage covers a large part of the structure or spans the major part of a primary load path. Hence the approach is likely to have most application in cases where the damage (in the form of microcracking) is restricted to specific regions of the structure with adjacent regions remaining unaffected. Knowledge (or at least an estimate) of the degraded laminate properties over the affected region does not in itself directly address whether or not the damage is tolerable. However it is important input to allowing an assessment of the effects of the damage that would consider further issues including the following.

(i) What influence does the affected region have on the load carrying capability of the structure as a whole?
(ii) Will the area of microcracking grow under normal operational loads?
(iii) Does the presence of the microcracking mean that the operating strains in the cracked region increase to above the design allowable values?

The above questions can be addressed by stress analysis taking into consideration the degraded properties of the affected region. A further question, relevant to cases of microcracking, but not addressed by analysis, is whether or not the cracking is expected to affect in any way the susceptibility of the laminate to environmental degradation.

4 Definition of a framework for defect assessment

4.1 Background

The preceding sections consider published methods for the analysis of defects that can be treated as delaminations or matrix microcracking. An important part of the development of a defect assessment procedure is the definition of how methods selected for inclusion should be implemented. This section considers briefly the framework for inclusion of various methods. A key feature of this framework is the selection of three levels for analysis, this following the approach of defect assessment procedures for metallic structures, see for examples [47-49], as discussed in an earlier review [50].

The first level aims to provide simple, readily implemented, rules for assessing acceptability. This seeks to rely only on easily available data and on hand calculations. The ease of implementation in a Level 1 analysis is typically achieved by inclusion of a range of simplifying assumptions, all of which must be demonstrably conservative. The overall degree of conservatism in a Level 1 assessment is often high. A further point worth noting is that the focus on simplicity often means that Level 1 analyses are not applicable to certain geometric features adjacent to which the stress state may be quite complex.

The second level aims to reduce the conservatism but still make for relatively straightforward implementation. The conservatism is reducing by application of more advanced analysis techniques, improved definition of defect parameters (perhaps determined by a more advanced inspection technique), and additional requirements on definition of material properties.

The third and final level reduces conservatism to a minimum but at the expense of often significant additional demands in terms of analysis and knowledge of defect parameters and material properties.

Following the review of available analysis methods the framework summarised in the sections that follow is suggested for the defect assessment procedure to be developed.

4.2 Delaminations

4.2.1 Level 1

For the Level 1 analysis of delaminations it is proposed that simple geometric criteria can be applied. These can be related to or derived on the basis of:

(i) The as-manufactured acceptability criteria. Such criteria are available in a variety of construction codes [50] for specific types of structure.
(ii) The design basis for the structure where this includes a consideration of damage tolerance criteria.
(iii) The results of analysis on a range of situations to establish a set of worst-case criteria.

4.2.2 Level 2

The Level 2 approach for delaminations will initially depend on the use of approximations based on analytical strain energy release rate solutions for beams. Where possible correction factors, considering the differences between the simple beam geometry and the actual geometry, will be derived using the results of finite element analysis.

The Level 2 approach will also allow the use of alternative estimates for strain energy release rate, e.g. based on interpolation of results from finite element models on similar structures. It will include a set of criteria defining minimum requirements for solutions that can be used.

Defect acceptability at Level 2 will most likely be determined by comparison to the threshold conditions for growth.

4.2.3 Level 3

The Level 3 approach for delaminations will rely primarily on estimation of strain energy release rates by use of detailed finite element modelling. The defect assessment procedure will provide guidance on how such analyses should be conducted.

Defect acceptability at Level 3 will, where appropriate, allow for operation beyond the growth threshold.

4.3 Matrix Microcracking

4.3.1 Level 1

Level 1 analysis of matrix microcracking will be based on simple geometric criteria related to the extent and location of the affected region. These will be based on as-manufactured acceptability criteria where appropriate and on the results of analysis for a range of situations aimed at establishing a set of worst-case criteria.

The potential effects of cracking on susceptibility to environmental degradation will be considered from Level 1 onwards.

4.3.2 Level 2

Level 2 analysis of microcracking will be based on available analytical stress solutions, using conventional laminated plate theory but taking into consideration fully degraded ply properties.

The acceptability of defects at Level 2 will be based on ensuring that growth of the affected region can not occur and that reduction in structural strength is below some limit.

4.3.3 Level 3

Level 3 analysis of microcracking will make use of more accurately calculated laminate properties based on detailed microcracking analysis. It will allow for detailed structural stress analysis by finite element methods (taking into consideration the degraded laminated properties over the affected region).

The acceptability of defects at Level 3 will be based primarily on ensuring that the reduction in structural strength remains below a defined limit. Growth of the region of microcracking will be allowed provided accurate analysis shows it is stable and on a timeframe that allows for monitoring up until the time at which the structural strength based criterion is approached.

4.4 Material properties required to perform the assessments

This report has focussed primarily on methods for predicting material parameters (such as stress, strain energy release rate or microcrack density) that are influenced by the applied loading and nature of the damage/defects. Acceptability of the defects is usually determined by comparison of these parameters to some intrinsic material property or defined allowable. Clearly, successful assessment depends as much on knowledge of these properties as it does on the ability to estimate parameters such as strain energy release rate. A major focus of the MMS 13 project, dealt with primarily in Task 2, is on defining test methods for determining the properties required. These must meet the needs of the methods for assessing acceptability. The requirements will vary according to the type of defects of concern and the Level of assessment. At this stage a preliminary definition of the requirements for each assessment level is provided. This is based on current expectations of how the assessment routes will be set up and will require refinement as the procedure is developed.

4.4.1 Level 1

In order to make for straightforward practical implementation, assessment at Level 1 should (i) attempt, as far as possible, not to include any reference to material properties and (ii) where material properties are needed in the assessment it should aim to allow the use generic values for the material type under consideration or design values (if known). Hence Level 1 requirements are not, at this stage, seen as having any impact on the types of testing to be carried out in MMS 13.

4.4.2 Level 2

Level 2 analysis for both delaminations and matrix microcracking will depend on knowledge (or estimates) of the ply base properties (from which laminate properties can be predicted). The properties required would be selected from the lists below.

Elastic properties
Longitudinal Young's modulus
Transverse Young's modulus
In-plane shear modulus
Major Poisson's ratio

Strength properties
Longitudinal tensile strength and strain to failure
Longitudinal compressive strength
Transverse tensile strength*
Transverse compressive strength
In-plane shear strength
Through thickness tensile strength
Through thickness compressive strength
Interlaminar shear strength
*Only when this is of value in assisting with the prediction of laminate strengths

Fracture energy properties
Threshold delamination strain energy release rate (Mode I)
Threshold delamination strain energy release rate (Mode I)
Critical delamination strain energy release rate (Mode I)
Critical delamination strain energy release rate (Mode II)

In cases where thermal loads are significant (e.g. laminates in structures subject to large temperature changes), the coefficients of expansion may be required.

In addition, knowledge of the fibre volume fraction may be useful in certain circumstances in that it allows some of the properties listed above to be predicted (provided the matrix and fibre properties are known).

In order to make for practical implementation, the Level 2 analysis should seek to minimise the requirements that measured properties for the material under consideration be used. Where possible allowance should be made for using conservative generic or design values. At the same time the user should be allowed to take advantage of measured properties where these are available. Generally, the use of measured properties would allow some of the conservatism needed in the estimated properties to be removed and hence result in a more favourable assessment. If defects determined by an initial Level 2 assessment to be unacceptable the option then remains for the user to determine some of the properties by test and this may then allow the defects to be justified as acceptable. Guidance will be given on how to determine those properties to which the assessment results are most sensitive and hence how best to select which properties might best be determined by measurement.

4.4.3 Level 3

The Level 3 analysis will make use of the same properties listed above for Level 2 but further properties may be required for specific analyses. A comprehensive list of additional properties is not provided here since it would depend on the details of the analysis being performed. However, examples of such properties include the through thickness elastic properties, critical strain energy release rate for transverse cracking and delamination fatigue crack growth parameters for modes I and II (see equations 4 and 5).

Even at Level 3 the analysis assessment approach should seek to allow the use of generic or design data where possible and appropriate. However, at this level the benefits of using test data are likely to be significant (particularly for the fracture parameters) and the additional cost/time is more likely to be justifiable.

4.5 The role of sub-structure testing

This report focuses on assessment primarily by analysis. A number of limitations within the various analysis/modelling methods have been highlighted and a detailed Level 3 analysis will usually involve significant complexity. Hence assessments by analysis will often benefit from support and validation by testing to increase confidence in the results (particularly where acceptability is not attained by a large margin). This type of approach, making use of modelling and testing in parallel, is commonly adopted in the aircraft industry for determining damage acceptability criteria for different structural components. The procedures to be developed should therefore consider inclusion of guidance on when support by testing might be appropriate and the types of tests that would most effectively support and/or validate the modelling.

5 Conclusion and recommendations

Published methods for the analysis of delaminations and matrix microcracking have been reviewed with the aim of determining how they can be incorporated in a practical defect assessment procedure.

Sophisticated modelling methods are available for both delaminations and microcracking. Implementation of these methods requires significant skill and effort on the part of the analyst and, in certain cases, may depend on generation of materials data by testing. These methods will form the basis for definition of the Level 3 assessments.

A greater challenge lies in defining the methods for implementation at Level 2. This level of assessment should be based on readily implemented techniques that do not require extensive accurate data input. In the case of delaminations the most promising approach appears to be through application of analytical solutions for simple geometries to more complex geometries, with empirically derived correction factors included if necessary. In the case of matrix microcracking conservative assumptions considering maximum degradation in matrix dominated properties may offer a practical approach.

The Level 1 assessment should, where possible, be based on simple geometric criteria. These will be determined, where possible, by consideration of existing as-manufactured defect acceptability criteria. Analytical methods for defect assessment are relevant in this regard however in that they may be needed to assess and validate the criteria finally included for use at Level 1.

The key actions required, within Task 5 of MMS 13, to complete definition of the assessment methods for the procedural guide are as follows.

- Evaluation of beam solutions for strain energy release rate as applied to plate and shell cases.

- Development of correction factors to reduce conservatism in delaminated beam solutions when applied to plate and shell cases.

- Evaluation of the use of fully degraded matrix dominated properties in determining laminate properties for damaged regions.

- Development of criteria for acceptable overall structural strength reductions associated with matrix microcracking.

- Definition of material classes and associated generic lower bound material properties.

- Development of Level 1 criteria for delaminations and matrix microcracking based on available as-manufactured criteria and supported by analysis results where needed.

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